JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 43, No. 1, January–February 2006

Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes should not exceed 2500 words (where a figure or table counts as 200 words). Following informal review by the Editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Simple Approach to Orbital Control Using Spinning Electrodynamic Tethers

Paul Williams*
Royal Melbourne Institute of Technology, Bundoora East, Victoria 3083, Australia

Introduction

ELECTRODYNAMIC tethers provide an extremely efficient means for performing orbit transfers. The technology behind electrodynamic tethers has only emerged in recent years and will most likely become the propulsive technology of choice on future generation spacecraft intended for low Earth orbit[1] or possibly even in orbit around Jupiter.[2] Electrodynamic-tether technology essentially allows systems to be transferred from one orbit to another by simply modulating the level of electric current in the long, conducting tether. In addition to generating a propulsive force, the resultant force is typically offset from the center of mass of the system, and hence a torque that causes the system to either librate (or spin) around the local vertical is generated. Furthermore, tethers are essentially perfectly flexible wires, and the presence of a distributed load along the tether must be carried by a reasonably large tether curvature. Variations in the planetary magnetic field and ionospheric plasma introduce lateral oscillations that are usually unstable and must be controlled. Although all of these important issues must ultimately be addressed in a control law for maneuvering an electrodynamic tether system, the focus of this Note is on the orbital control aspects of such a system.

Ishige et al.[3] studied the possibility of using an electrodynamic-tether system for debris mitigation by maneuvering an electrodynamic tether system to rendezvous with debris, deploying the captured debris with the tether, and then changing the system orbit so that the debris can be safely released on a reentry trajectory. Strategies for maneuvering the system using simple trigonometric functions of the argument of latitude were presented. Yamagiwa et al.[4] studied the performance of an electrodynamic tether for changing orbital altitude of a spacecraft with a hanging tether. Comparisons in terms of mission times for orbital transfers on different inclination orbits were performed with an ion thruster. They concluded that the EDT concept is less useful if the inclination is greater than about 30 deg. However, this does not take into account more general orbital maneuvers or other system configurations. Lanoix et al.[5] studied the effect of both insulated and bare tethers for deorbiting applications. They assumed that the tether remains straight in their analysis, but is allowed to extend longitudinally, and used a simple control law to help stabilize the tether librations. The control current is the superposition of a constant current for deorbit and a current that varies with the in-plane libration angle. Tragesser and San[6] developed a guidance algorithm for maneuvering hanging electrodynamic tethers between arbitrary orbits (restricted, however, to nonzero eccentricities because of the restrictions imposed by the perturbation equations). In their approach, the tether librations were neglected, and control of the orbital elements was achieved by determining the variation in current to produce secular changes in each of the orbital elements. A total control law was constructed by a linear combination of the individual control laws and solving for the combination of coefficients to give the desired change in the orbital elements. To account for possible restrictions on current levels, the time of flight was adjusted to keep the current within the required bounds. The technique works reasonably well, with some small residual errors in the orbital elements caused by neglecting the periodic effects in the orbital elements. Williams[7] recently used direct transcription to solve an optimal orbit boost/deboost problem with a librating tether. In this case, the variation in orbital parameters is not secular, and the tether librations were shown to be beneficial because they allow greater components of the electrodynamic force to be directed along the velocity vector. Pearson et al.[8] summarized a proposed concept for an electrodynamic tether-driven spacecraft called the ElectroDynamic Delivery Express (EDDE) whose purpose is to deliver multiple small satellites from low Earth orbit (LEO) to different LEO orbits without expending propellant. They studied both hanging and spinning tether configurations for performing orbital maneuvers and concluded that spinning tethers offer advantages from the point of view of efficiency and stability of the tether lateral modes.[9] Control of the flexible tether modes is achieved by superimposing an additional electromotive force (emf) over the drive emf to suppress the unwanted lateral vibration modes.

The purpose of the current Note is to develop simple guidance equations for spinning electrodynamic tethers that can be used to provide approximate closed-loop guidance for orbital transfers. The effect of using spinning electrodynamic tethers will be shown to be beneficial compared to using hanging tethers for general orbital maneuvers.

Mathematical Model

The motion of the tether system center of mass is governed by the set of general perturbation equations. In Gauss's form, they are[10]

$$\dot{a} = (2a^2/h)[e\sin(\nu) f_r + (p/r) f_t] \tag{1}$$

$$\dot{e} = (1/h)\{p\sin(\nu) f_r + [(p+r)\cos(\nu) + re] f_t\} \tag{2}$$

$$\dot{\omega} = (1/he)[-p\cos(\nu) f_r + (p+r)\sin(\nu) f_t] - [r\sin(\omega+\nu)\cos i/h\sin i] f_h \tag{3}$$

$$\dot{i} = [r\cos(\omega+\nu)/h] f_h \tag{4}$$

$$\dot{\Omega} = [r\sin(\omega+\nu)/h\sin i] f_h \tag{5}$$

$$\dot{\nu} = h/r^2 + (1/eh)[p\cos(\nu) f_r - (p+r)\sin(\nu) f_t] \tag{6}$$

Received 13 March 2005; revision received 5 June 2005; accepted for publication 13 July 2005.

*Research Fellow, P.O. Box 71, School of Aerospace, Mechanical and Manufacturing Engineering; paul.williams@rmit.edu.au.

where a is the orbit semimajor axis; $h=\sqrt{[\mu a(1-e^2)]}$ is the orbit angular momentum; ν is the orbit true anomaly; ω is the argument of perigee; $p=a(1-e^2)$ is the semilatus rectum; r is the orbit radius; e is the orbit eccentricity; Ω is the right ascension of the ascending node; i is the orbit inclination; and f_r, f_t, and f_h are the components of the disturbing acceleration vector in the radial, transverse, and orbit normal directions, respectively,

$$f_r=(IL/m)(B_z\sin\theta\cos\phi-B_y\sin\phi) \tag{7}$$

$$f_t=(IL/m)(B_x\sin\phi-B_z\cos\theta\cos\phi) \tag{8}$$

$$f_h=(IL/m)(B_y\cos\theta\cos\phi-B_x\sin\theta\cos\phi) \tag{9}$$

The components of the magnetic field vector (assumed to be a non-tilted dipole) in the local orbital frame are defined by[11]

$$B_x=-2\left(\mu_m/r^3\right)\sin(\omega+\nu)\sin i \tag{10}$$

$$B_y=\left(\mu_m/r^3\right)\cos(\omega+\nu)\sin i \tag{11}$$

$$B_z=\left(\mu_m/r^3\right)\cos i \tag{12}$$

where μ_m is the magnetic moment of the Earth's dipole. Equations (3), (5), and (6) contain singularities for $e=i=0$, which restricts the following development to nonzero eccentricities or inclinations if the argument of perigee or longitude of the ascending node are to be controlled directly. The variation of the true anomaly given by Eq. (6) is not used in the development of the guidance equations, but is used in their validation.

Guidance Equations

The idea of the guidance scheme is to find a form of the current law that yields secular changes in each of the orbital elements. Naturally, the orbital elements are coupled so that control applied to one will lead to variations in the others. The assumptions for controlling a spinning tethered system are that the in-plane libration angle varies as a function of the true anomaly according to $\theta=s\nu$, where s is the approximate spin rate in revolutions per orbit, and that the out-of-plane angle is negligible, $\phi\approx 0$. This could be achieved by using a self-balanced tether concept.[12] In an idealized setting, if the current is constant along the tether length as in the insulated case then a configuration with equal end masses would yield a zero net torque on the tether caused by electromagnetic forces. The situation is more complex if bare tethers are used, and the self-balanced concept warrants further investigation. The use of a spinning tether can create some additional guidance and control problems that are not prevalent in hanging tethers. The main issue is creating and maintaining the desired tether spin rate. Spin-up could be achieved by "pumping" the tether librations until the system commences rotation, as discussed by Carroll.[13] The spin rate can be maintained by periodically reeling the tether as a function of the orbit true anomaly, eccentricity, and desired tether spin rate. Any additional perturbations to the tether librations would need to be controlled either via tether reeling or by the superposition of an additional control current over the current for orbital maneuvers. Because the effects of changes in tether length and additional control current would in most cases be small, their influence on the mean orbital motion has been neglected in the ensuing analysis. However, detailed implementation of such a system would demand that these aspects be integrated fully into the controller.

If it is assumed that the orbit eccentricity is small, then we can expand the orbit perturbation equations for small eccentricities and keep the dominant terms. The resulting expressions are integrated to give the changes in the orbital elements. As an approximation, the orbital elements on the right-hand side of the perturbation equations are assumed constant for the integration. The resulting set of equations describing the change in orbit elements are as follows:

$$\Delta a\approx-\frac{2L\mu_m\cos i}{mna^3}\int_{t_0}^{t_f}I\cos(s\nu)\,\mathrm{d}t \tag{13}$$

$$\Delta e\approx\frac{L\mu_m\cos i}{mna^4}\int_{t_0}^{t_f}I[\sin\nu\sin(s\nu)-2\cos\nu\cos(s\nu)]\,\mathrm{d}t \tag{14}$$

$$\Delta\omega\approx-\frac{L\mu_m\cos i}{mna^4}\int_{t_0}^{t_f}\frac{I[\cos\nu\sin(s\nu)+2\sin\nu\cos(s\nu)]}{e\,\mathrm{d}t} \tag{15}$$

$$\Delta i\approx\frac{L\mu_m\sin i}{mna^4}\int_{t_0}^{t_f}I\{\cos^2(\nu+\omega)\cos(s\nu)$$

$$+\sin[2(\nu+\omega)]\sin(s\nu)\}\,\mathrm{d}t \tag{16}$$

$$\Delta\Omega\approx\frac{L\mu_m}{mna^4}\int_{t_0}^{t_f}I\Bigg\{\sin(s\nu)-\sin(s\nu)\cos[2(\nu+\omega)]$$

$$+\frac{1}{2}\sin[2(\nu+\omega)]\cos(s\nu)\Bigg\}\,\mathrm{d}t \tag{17}$$

The form of the current control law is determined so that secular changes to the orbit elements in Eqs. (13–17) are produced. The particular selections for this work are

$$a:\ \cos(s\nu)$$

$$e:\ \cos\nu\cos(s\nu)-\sin\nu\sin(s\nu)$$

$$\omega:\ \sin\nu\cos(s\nu)+\cos\nu\sin(s\nu)$$

$$i:\ \cos[2(\nu+\omega)]\cos(s\nu)+\sin[2(\nu+\omega)]\sin(s\nu)$$

$$\Omega:\ \sin[2(\nu+\omega)]\cos(s\nu)-\cos[2(\nu+\omega)]\sin(s\nu) \tag{18}$$

Note that other selections might be more efficient, but for the purposes of this study the preceding selections are acceptable. Hence, the control law can be constructed in the form

$$I=X_1\cos(s\nu)+X_2[\cos\nu\cos(s\nu)-\sin\nu\sin(s\nu)]$$

$$+X_3[\sin\nu\cos(s\nu)+\cos\nu\sin(s\nu)]$$

$$+X_4\{\cos[2(\nu+\omega)]\cos(s\nu)+\sin[2(\nu+\omega)]\sin(s\nu)\}$$

$$+X_5\{\sin[2(\nu+\omega)]\cos(s\nu)-\cos[2(\nu+\omega)]\sin(s\nu)\} \tag{19}$$

where X_j, $j=1,\ldots,5$ are coefficients to be determined based on the flight time and desired change in the system orbit. Note that for the case of a hanging tether $s=0$ and the control law reduces to the one presented in Ref. 6.

The particular form of the current is determined by the coefficients in Eq. (19). These coefficients are determined by evaluating the integrals in Eqs. (13–17) over one orbit. It is necessary to change the independent variable from time to true anomaly by the following relationship:

$$\mathrm{d}t=\sqrt{p^3/\mu}\,[1/(1+e\cos\nu)^2]\,\mathrm{d}\nu \tag{20}$$

This expression is substituted into the original perturbation equations, and the resulting expressions are expanded to first order in the eccentricity. After substituting the expression for the control current into the equations and integrating over one orbit, a system of equations linear in X_j is obtained. For the cases in which $s\in\mathbb{N}>2$, then the form of coefficients of X_j in the equations are independent of the system spin rate:

$$\begin{bmatrix} -2\pi\Upsilon\cos i & -3\pi e\Upsilon\cos i & 0 & 0 & 0 \\ -3\pi e\Upsilon\cos i/2a & -3\pi\Upsilon\cos i/2a & 0 & 0 & 0 \\ 0 & 0 & -(12/e+15e)\Upsilon\pi\cos i/8a & 0 & -3\pi\Upsilon\cos i/4a \\ \pi\Upsilon\sin i/2a & 0 & 0 & 3\pi\Upsilon\sin i/a & 0 \\ 0 & 0 & 0 & 0 & 3\pi\Upsilon/4a \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = \begin{bmatrix} \Delta a \\ \Delta e \\ \Delta\omega \\ \Delta i \\ \Delta\Omega \end{bmatrix} \quad (21)$$

where $\Upsilon = L\mu/\mu_m$. The form of the equations for the hanging tether case can be derived in a similar manner.

Because the changes in the orbital elements are achieved by secular changes, a spinning tether is unlikely to be more efficient than a hanging tether for changing the orbital parameters that are most effectively changed by tangential thrusting, such as the semimajor axis and eccentricity. If the current control is determined by other means such as Pontryagin's maximum principle, then a spinning tether can be more efficient than a hanging tether even for changes in the semimajor axis.[14]

Numerical Results

For comparative purposes, five different orbit-transfer scenarios are considered with the system at different initial orbit inclinations in the range $10 \leq i \leq 70$ deg. The tether length is assumed to be 15 km, and the total system mass is 420 kg. A set of common initial orbital parameters is used for the first four cases: $[a, e, \omega, \Omega]_0 = [6878$ km, 0.02, 30 deg, 30 deg$]$. The first case is a simple increase in semimajor axis with $\Delta a = 200$ km, the second is an increase in eccentricity with $\Delta e = 0.18$, the third is an increase in orbit inclination with $\Delta i = 10$ deg, and the fourth is a rotation of the longitude of the ascending node with $\Delta\Omega = 10$ deg. The final case is a general orbit transfer with $[a, e, \omega, \Omega]_0 = [6878$ km, 0.02, 20 deg, 70 deg$]$ and $[\Delta a, \Delta e, \Delta\omega, \Delta i, \Delta\Omega] = [303$ km, 0.005, 6 deg, 1 deg, 8 deg$]$. Numerical estimates for the required flight with the maximum current limited to 4 A were calculated over a range of initial orbital inclinations. Figure 1 summarizes the results showing the difference between the hanging tether transfer and the spinning tether transfer times. (A negative ordinate implies a faster transfer with a hanging tether.) This illustrates the dependency of the flight time on the initial orbit inclination. The most significant advantage of using spinning tethers occurs for inclination changes at low initial orbit inclinations (savings on the order of 500 orbits). Savings in the transfer time on the order of 100 orbits are possible for changes in the ascending node, which is independent of the initial inclination caused by the nontilted dipole model, whereas the general orbit transfer produces savings of between 84 and 52 orbits. The efficiency of the hanging tether is only slightly superior to a spinning tether for changing the semimajor axis. Finally, changes in the eccentricity appear to be more efficient with a hanging tether than with a spinning tether using the guidance algorithm presented here.

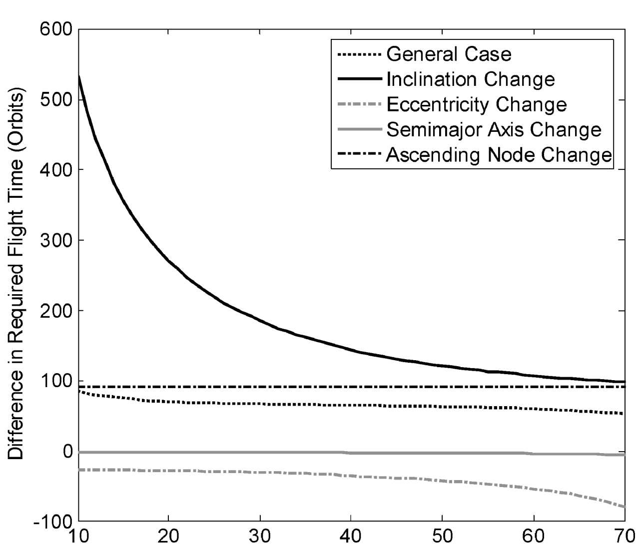

Fig. 1 Flight time difference for spinning (S) and hanging tether (HT) orbit transfer (HT–S) vs initial orbit inclination.

The initial values of the control coefficients and number of orbits are given in Table 1 for a specific case of the general orbit transfer ($i_0 = 55$ deg). The control coefficients are remarkably similar for both cases despite the differences in the form of the control law.

Table 1 Initial control coefficients and flight time for general orbit change

Parameter	Hanging tether	Spinning tether
Orbits required	255.66	189.30
X_1	−0.234	−0.632
X_2	−0.0611	−0.0834
X_3	−0.0493	−0.0710
X_4	0.989	0.891
X_5	3.418	3.077

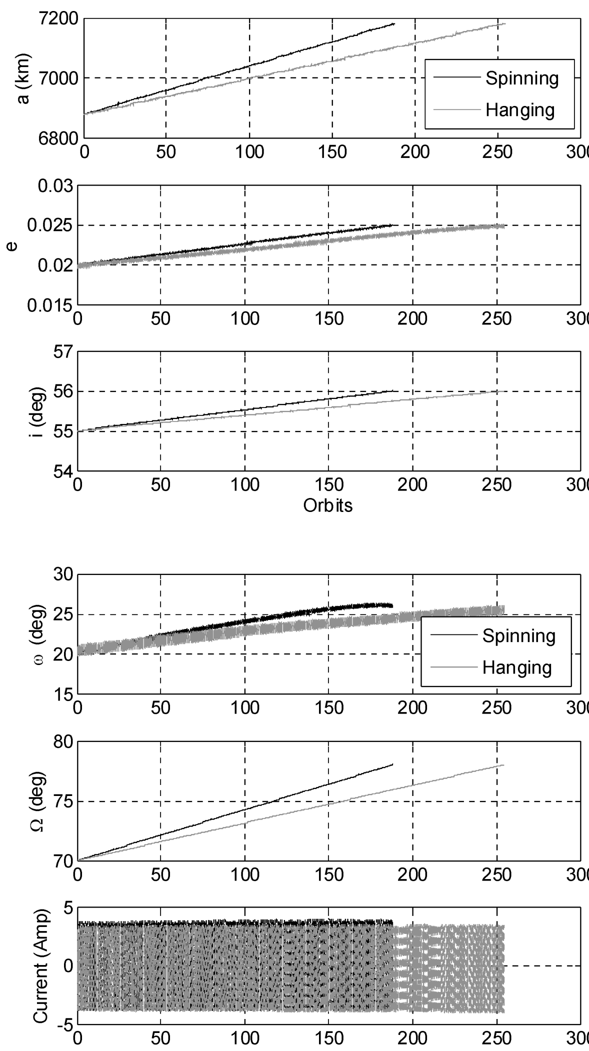

Fig. 2 General orbit change with spinning and hanging ED tethers.

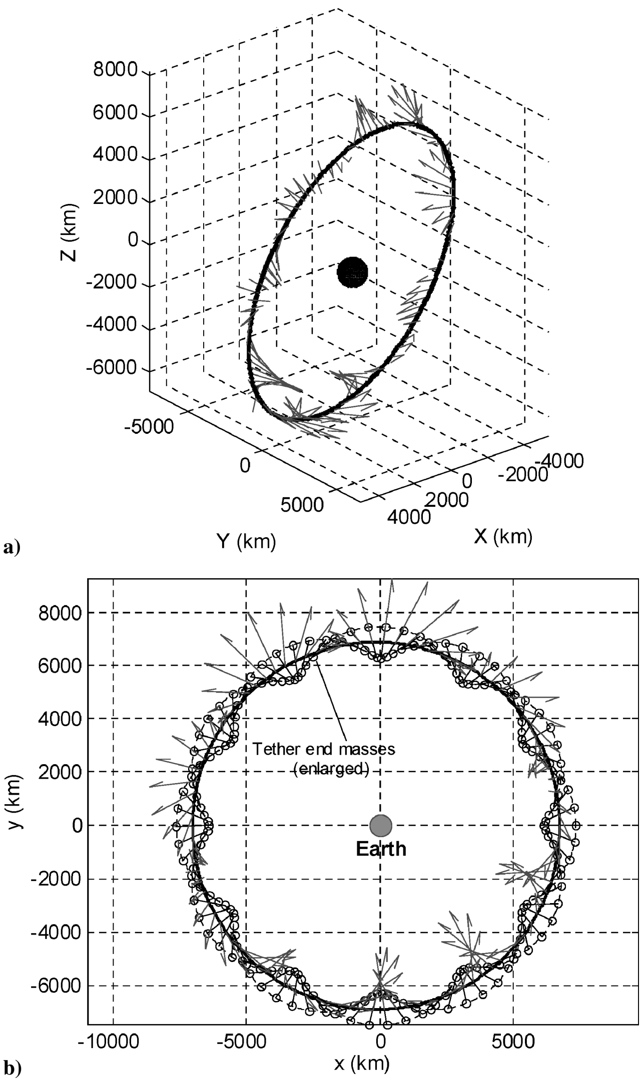

Fig. 3 Variation of ED force over an orbit with spinning tether (Earth not to scale): a) orbit in geocentric equatorial coordinates and b) perifocal coordinates.

The control of the longitude of the ascending node dominates both controllers as indicated by the large value of the coefficient X_5. For a detailed simulation, the control coefficients are updated regularly, approximately once per orbit, to compensate for the time variation of the orbital elements.

Numerical results for the variation in orbital elements are shown in Fig. 2, where the mean linear variation of each of the orbital elements is evident. The variation of the argument of perigee deviates somewhat from a straight line, suggesting the need for feedback. The short-term periodic variations of the orbital elements appear to be more dominant for the hanging tether case than for the spinning case, as can be seen by comparing the plots for eccentricity and argument of perigee. The fact that the spinning electrodynamic (ED) tether can utilize Lorentz forces over a much wider range of directions can be seen by examining the control current for the hanging and spinning cases. The current for the spinning case oscillates between the maximum permissible values of the current, whereas in the hanging tether case the current only reaches its maximum on the lower bound. Hence, the spinning tether case can more efficiently utilize the available current to generate the required control forces. Figure 3 shows a snapshot of the orbit and ED force directions over one orbit. The complex variation of the total electromagnetic force over an orbit can be seen to be a function of the tether orientation as well as the inertial position of the tether system. The errors in the final elements are of comparable magnitudes for this particular simulation.

Hanging:

$$\Delta a = -0.017 \text{ km}, \qquad \Delta e = 0.0000014, \qquad \Delta\omega = 0.0026 \text{ deg}$$

$$\Delta i = -0.000003 \text{ deg}, \qquad \Delta\Omega = -0.000038 \text{ deg}$$

Spinning:

$$\Delta a = 0.0092 \text{ km}, \qquad \Delta e = -0.0000013, \qquad \Delta\omega = 0.010 \text{ deg}$$

$$\Delta i = 0.000053 \text{ deg}, \qquad \Delta\Omega = -0.00014 \text{ deg}$$

Conclusions

A guidance scheme based on secular variations in the orbital elements, presented previously, has been extended to the case involving spinning tethered systems. A spinning tether system can utilize the electrodynamic Lorentz forces more effectively than a hanging or librating tether for maneuvers that require out-of-plane thrust forces, and hence, for similar current levels can more quickly perform general orbital maneuvers. The added stability created by the centrifugal stiffening from the rotation of a flexible tether system makes spinning an attractive method for enhancing electrodynamic tether systems. More work is required to develop control methods for stabilizing the tether rotation.

References

[1] Johnson, L., Estes, R. D., Lorenzini, E., Martinez-Sanchez, M., Sanmartin, J., and Vas, I., "Electrodynamic Tethers for Spacecraft Propulsion," AIAA Paper 98-0983, Jan. 1998.

[2] Gallagher, D. L., Johnson, L., Moore, J., and Bagenal, F., "Electrodynamic Tether Propulsion and Power Generation at Jupiter," NASA TP-1998-208475, June 1998.

[3] Ishige, Y., Kawamoto, S., and Kibe, S., "Study on Electrodynamic Tether System for Space Debris Removal," International Astronautical Federation, IAF-02-A.7.04, Oct. 2002.

[4] Yamagiwa, Y., Sakata, Y., and Hara, N., "Performance of Electrodynamic Tether Orbit Transfer System on the Orbit with Inclination," *Proceedings of the 22nd International Symposium on Space Technology and Science*, Vol. 2, Japan Society for Aeronautical and Space Sciences and 22nd ISTS Organizing Committee, Tokyo, 2000, pp. 1807–1812.

[5] Lanoix, E. L. M., Misra, A. K., Modi, V. J., and Tyc, G., "Effect of Electromagnetic Forces on the Orbital Dynamics of Tethered Satellites," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 6, 2005, pp. 1309–1315.

[6] Tragesser, S. G., and San, H., "Orbital Maneuvering with Electrodynamic Tethers," *Journal of Guidance, Control, and Dynamics*, Vol. 26, No. 5, 2003, pp. 805–810.

[7] Williams, P., "Optimal Orbital Transfer with Electrodynamic Tether," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, 2005, pp. 369–372.

[8] Pearson, J., Carroll, J., Levin, E., Oldson, J., and Hausgen, P., "Overview of the Electrodynamic Delivery Express (EDDE)," AIAA Paper 2003-4790, July 2003.

[9] Pearson, J., Levin, E., Carroll, J. A., and Oldson, J. C., "Orbital Maneuvering with Spinning Electrodynamic Tethers," AIAA Paper 2004-5715, Aug. 2004.

[10] Kechichian, J. A., "Trajectory Optimization Using Nonsingular Orbital Elements and True Longitude," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 5, 1997, pp. 1003–1009.

[11] Pelaez, J., Lorenzini, E. C., Lopez-Rebollal, O., and Ruiz, M., "A New Kind of Instability in Electrodynamic Tethers," American Astronautical Society, Paper 00-190, Jan. 2000.

[12] Pelaez, J., "Self Balanced Electrodynamic Tethers," AIAA Paper 2004-5309, Aug. 2004.

[13] Carroll, J. A., "Preliminary Design for a 1 Km/Sec Tether Transport Facility," NASA Office of Aeronautics and Space Technology, Third Annual Advanced Propulsion Workshop, Jan. 1992.

[14] Williams, P., "Optimal Orbital Maneuvering Using Electrodynamic Tethers," American Astronautical Society, Paper 05-206, Jan. 2005.

I. Vas
Associate Editor